value. The results of two of the calibration tests are shown in Fig. 1. The first test was conducted with the pressure at 7 psia whereas the second test was conducted at atmospheric pressure.

Although the results were positive and the technique was considered usable, it should be noted that a certain inherent uncertainty remains in the mass flow rate measurements because of the small differences in the thermal conductivities and viscosities. . Given a particular signal voltage, the error introduced by the difference between the corresponding mass flow rates for pure Freon 22 and pure Freon 114 increases from about $\pm 1.6\%$ at the lowest Reynolds number measurements to as much as $\pm 4\%$ with $Re \simeq 2$.

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Hypersonic Lee-Surface Heating Alleviation on Delta Wing by Apex-Drooping

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In a recent Note,[1] Whitehead reported high heating rates on the leeward meridian of a 75° swent delta wing at 5° incidence in $M = 6$ flow. This peak heating was originally thought to be associated with flow impingement induced by free vortices following separation from the sharp leading-edges (Fig. 1a). Subsequently, a detailed study was made of a number of lee-surface oil-flow patterns obtained in tests representative of conditions where Ref. 1 had shown the first peak in leeward heating to be most pronounced. It was found that the flow remained attached over the apex region and was initially vortex-free, the "feather" pattern characteristic of a vortex system appearing only at some distance downstream of the apex (Fig. 2 shows a typical pattern). This observation led to the conjecture that the vortices in question originate within the laminar boundary layer as a result of three-dimensional flow development in the apex region (Fig. 1b). If so, by preventing vortex formation in this region, a vortex-free flow over the entire wing should result. Since such a vortex system has not been observed on delta wings at zero incidence, drooping the apex portion of the wing to locally align it with the freestream could be a possible means of vortex suppression. This reasoning formed the basis of a brief experimental investigation with a 75° delta wing at 5° incidence in the 11-in. Hypersonic ($M = 6.8$) Blowdown Tunnel at NASA Langley Research Center, at a unit Reynolds number of 0.2 million/in. (model length Reynolds number of 2 million).

Preliminary oil-flow tests showed that apex-drooping successfully eliminated the high shear and vortex-associated feather pattern found on the delta wing without droop. In order to confirm the beneficial effect of vortex suppression on the centerline heating, experiments on a heat-transfer model using the phase-change paint method were carried out. In this method,[2] the model is coated with a paint of known melting point and then exposed to the tunnel flow; the time increment to reach the melting temperature at a given model location measures the local heat-transfer rate.

The heat-transfer model, incorporating a silicone-rubber insert over a substantial portion of the upper surface to reduce heat conduction, was first tested in the flat condition (i.e., without droop), and then with the apex bent down approximately along a circular arc through 5° slope, as indicated in Fig. 3. A temperature-sensitive paint with a melting point of 103°F, the lowest available, was applied to the rubber surface. During the tests, the model was photographed with a 35-mm time lapse camera at 10 frames/sec.

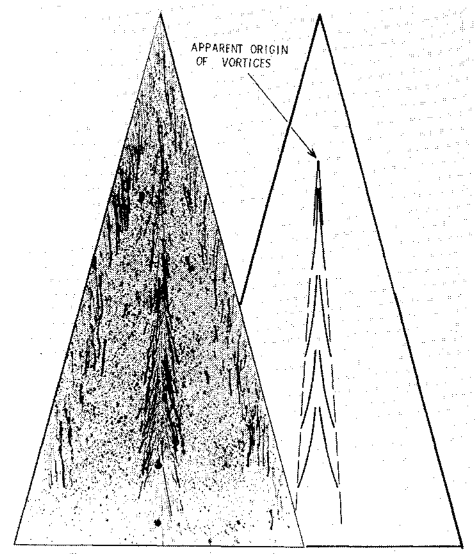

Fig. 2 Oil flow picture showing origin of vortices on lee-surface at 5° incidence (Reynolds number 0.2 million/in.).

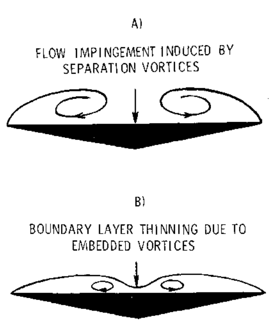

Fig. 1 Vortex flow on lee-surface of delta wing.

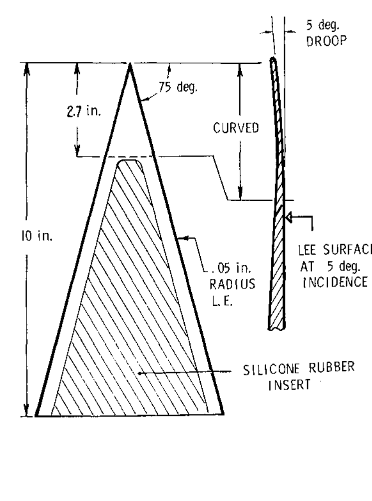

Fig. 3 Delta-wing heat-transfer model geometry.

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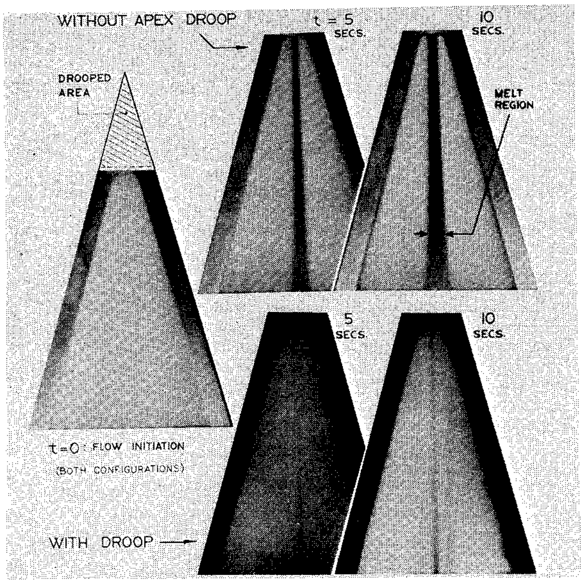

Fig. 4 Photographs showing thermo-sensitive paint melt patterns on delta wing with and without apex droop.

A comparison of the film records for the two-test configurations, at 5 and 10 sec (Fig. 4) shows that drooping the apex of the delta wing succeeded in greatly reducing the peak heating on the entire leeward centerline. The faint trace of vortices apparent on the droop-apex delta may result from a slight deflection of the model with wind-on, which induced a small leeward incidence at the apex. The peak heat-transfer rate associated with this weak vortex system is, however, estimated from film data to be less than one-third of the wing without droop.

Comparison with the data of Ref. 1, which showed the peak heat-transfer rate in the presence of vortices to be approximately three times the two-dimensional laminar boundary-layer calculation, suggests that the lee-surface heating on the droop-nose delta has been reduced to a level comparable with the two-dimensional laminar value. Interestingly, the spanwise variation of heat-transfer rate on a flat plate, reported in Ref. 3 and ascribed to longitudinal vortices generated within a laminar boundary layer, contain peaks similar in magnitude to those measured on the undrooped delta wing.

Together with the flow visualization studies mentioned earlier, the near-elimination of lee-meridian heating achieved by apex-drooping may be taken to favor the original conjecture that, at least at low incidence angles, the vortices arise as a result of cross flow within the laminar boundary layer.

A detailed analysis of the fluid-dynamic phenomenon in the apex region remains to be attempted; however, an intuitive approach may be taken, keeping in view the well-known two-layer model of the hypersonic laminar boundary layer,[4] consisting of a wall region of greatly reduced momentum flux and an outer layer. We consider the development of the boundary layer in a three-dimensional interaction in the region of rising pressure between the leading edge and the plane of symmtery, associated with the realinement of the inviscid streamlines initially bent inwards due to expansion of the freestream around the leading edge.[5]

The low momentum fluid, unable to penetrate the adverse pressure gradient, will be turned axially much earlier than the flow in the outer layer. This skewed boundary layer would then be expected to develop longitudinal vorticity concentrated at the junction of the two layers and in the region of the maximum pressure gradient on the wing surface. The resulting symmteric contrarotating pair of embedded vortices, much like the vortex pair following separation on delta wings, will act to drain the boundary layer away from the plane of symmtery, leading to increased shear and heat-transfer rate on the centreline.

A direct experimental confirmation of the existence of embedded vortices is desirable since they appear to be a plausible alternate mechanism to explain leeward heating peaks which until now have been exclusively related to separation vortices. The embedded vortex phenomenon, which might be common to three-dimensional boundary-layer flows on lifting bodies, appears worthy of investigation as it may hold the key to the practical problem of determining and controlling lee-side heating on hypersonic configurations.

In conclusion, it has been shown that the vortex-associated peak heating on the lee-side meridian of a delta wing at hypersonic speed can be practically eliminated by aligning the apex region with the freestream. The result suggests that the vortices are generated within the apex zone as a result of cross flow in the laminar boundary layer.

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Supersonic Nozzle Discharge Coefficients at Low Reynolds Numbers

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Nomenclature

$A(\gamma)$ = function of γ defined in Eq. (8)
$B(\gamma)$ = function of γ defined in Eq. (9)
C_D = discharge coefficient
d_t = nozzle throat diameter
$f(\gamma)$ = function of γ defined in Eq. (11)
M = Mach number in the nozzle isentropic core
Re = nozzle throat Reynolds number, $4\dot{W}_{\text{ideal}}/\pi d_t \mu_0$
r = nozzle radius
r_c = radius of curvature at the nozzle throat
r_t = nozzle throat radius
u = velocity in the boundary layer
u_s = slip velocity at the nozzle wall
U = velocity in the isentropic core
$\dot{W}_{\text{ideal}}$ = one-dimensional ideal flowrate through the nozzle
x = axial coordinate along the nozzle contour
y = coordinate measured normal to the nozzle contour
γ = ratio of specific heats
δ = boundary-layer thickness
δ^* = displacement thickness
η = dimensionless boundary-layer thickness, y/δ
θ = momentum thickness
μ_0 = viscosity in the nozzle stagnation chamber

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